05.23.17 СТРОИТЕЛЬНАЯ МЕХАНИКА

Определение напряжённо-деформированного состояния коротких внецентренно-сжатых трубобетонных колонн методом конечных элементов путём сведения трёхмерной задачи к двумерной

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Аннотация. В статье приводится вывод уравнений МКЭ, позволяющих производить расчёт коротких трубобетонных колонн в плоской упругой постановке. Работа стальной обоймы моделируется ферменными конечными элементами. Производится сопоставление полученных результатов с результатами вычислений с использованием объемных и оболочечных конечных элементов.

Ключевые слова: МКЭ, внецентренное сжатие, численные методы, трубобетонные колонны, бетон.

Introduction

In modern high-rise construction, steel-reinforced concrete structures are widely used, including structures consisting of a concrete core and a steel tube. More acute is the question of improving regulato-

DETERMINATION OF STRESS-STRAIN STATE OF SHORT ECCENTRICALLY LOADED CONCRETE-FILLED STEEL TUBULAR (CFST) COLUMNS USING FINITE ELEMENT METHOD WITH REDUCING THE PROBLEM FROM THREE-DIMENSIONAL TO TWO-DIMENSIONAL

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Abstract. The article presents the derivation of the FEM equations, which make possible to calculate short concrete-filled steel tubular columns in a plane elastic formulation. Steel tube is modeled by 1D bar finite elements. The obtained results are compared with the results of calculations using 3D solid and shell finite elements.
Keywords: FEM, eccentric compression, numerical methods, CFST columns, concrete.

ry documentation, increasing the economic efficiency and reliability of such structures based on new theoretical and experimental data. Numerical modeling of the CFST columns deformations in existing finite element software requires significant computing power and time. Correct accounting of the tube affect on the concrete core stress-strain state is possible only when using volumetric finite elements for concrete and shell ones for steel [1; 2].

In this work, a technique has been developed for converting the calculation of a short eccentrically compressed CFST column in an elastic formulation to a plane problem, which significantly reduces the dimension of the stiffness matrix used in the calculation, the required machine time and memory.

Derivation of resolving equations

We consider a short concrete-filled steel tubular column of unit length shown in Fig. 1, loaded with force F and moment M. Cross-section characteristics are shown in Fig. 2.

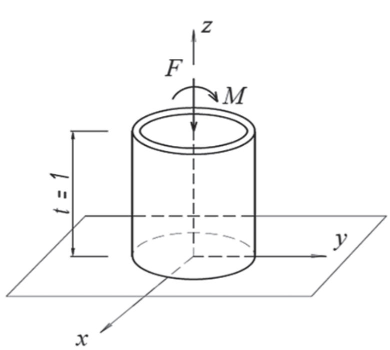

Fig. 1. Column loading scheme

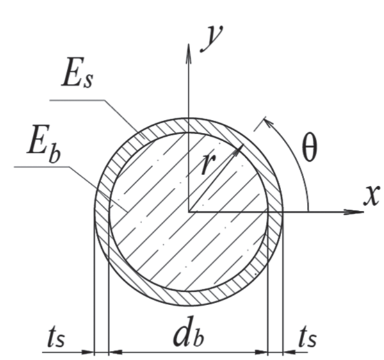

Fig. 2. Cross section of the column

According to the variational principle of the minimum total potential energy [3]:

$$\delta\Pi + \delta V = 0, \quad (1)$$

Π is the elastic strain energy, V is the potential of applied forces.

In the further derivation of the equations, it is assumed that the Navier's hypothesis is satisfied. Steel and concrete work together without sliding. Normal strain along z axis:

$$\varepsilon_z = \varepsilon_0 + y \cdot \chi_0, \quad (2)$$

ε_0 is the normal strain of the column's central longitudinal axis and

$$\chi_0 = -\frac{d^2 f_y}{dz^2}$$

is the curvature of the longitudinal central axis of the column, which are constant along the length.

For the case shown in Fig. 1 with the fixed support of the lower end of a unit length column, taking into account the direction of action of the load F:

$$\delta V = F\delta\varepsilon_0 + M\delta\chi_0 = \delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T \begin{Bmatrix}F\\ M\end{Bmatrix}. \quad (3)$$

The variation of the potential strain energy of the column is the sum of the strain energies variations of the concrete core and the steel tube:

$$\delta\Pi = \delta\Pi_b + \delta\Pi_S, \quad (4)$$

$$\delta\Pi_b = \int_{A_b}\left(\sigma_z\delta\varepsilon_z + \sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy}\right)dA, \quad (5)$$

A_b is the area of the concrete cross section part. Using the generalized Hooke's law, we get:

$$\int_{A_b}\sigma_z\delta\varepsilon_z dA = \int_{A_b}\left(\lambda_b\left(\varepsilon_z+\varepsilon_x+\varepsilon_y\right)+2\mu_b\varepsilon_z\right)\delta\varepsilon_z dA = $$
$$= \int_{A_b}\left(\lambda_b + 2\mu_b\right)\varepsilon_z\delta\varepsilon_z dA + \quad (6)$$
$$+\int_{A_b}\lambda_b\left(\varepsilon_x+\varepsilon_y\right)\delta\varepsilon_z dA = \delta\Pi_{b1} + \delta\Pi_{b2},$$

here λ_b, μ_b are Lamé parameters [4].

$$\int_{A_b}\left(\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}\right)dA=$$

$$\int_{A_b}\left(\begin{bmatrix}\lambda_b+2\mu_b & \lambda_b & 0\\ \lambda_b & \lambda_b+2\mu_b & 0\\ 0 & 0 & \mu_b\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}^T\cdot\delta\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}dA\right)+$$

$$+\int_{A_b}\lambda_b\begin{Bmatrix}\varepsilon_0+y\chi_0\\ \varepsilon_0+y\chi_0\\ 0\end{Bmatrix}^T\cdot\delta\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}dA=\delta\Pi_{b3}+\delta\Pi_{b4}. \tag{7}$$

$$\delta\Pi_{b1}=\int_{A_b}\left(\lambda_b+2\mu_b\right)[1\quad y]\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}[1\quad y]\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}dA=$$

$$=\int_{A_b}\left(\lambda_b+2\mu_b\right)\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T\begin{Bmatrix}1\\ y\end{Bmatrix}[1\quad y]\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}dA= \tag{8}$$

$$=\int_{A_b}\left(\lambda_b+2\mu_b\right)\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T\begin{bmatrix}1 & y\\ y & y^2\end{bmatrix}\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}dA.$$

Transposing the right-hand side of expression (8), and taking into account that the values of ε_0, χ_0 and their variations are constant over the cross-section, we get:

$$\delta\Pi_{b1}=\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T\int_{A_b}\left(\lambda_b+2\mu_b\right)\begin{bmatrix}1 & y\\ y & y^2\end{bmatrix}dA\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}. \tag{9}$$

When modeling the concrete core, we will use triangular finite elements. The contribution of a single finite element to the total part of the potential energy variation $\delta\Pi_{b1}$ will be:

$$\delta\Pi^e_{b1}=\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T\left(\lambda_b+2\mu_b\right)\begin{bmatrix}1 & y_m\\ y_m & y_m^2\end{bmatrix}\cdot\Delta^e\cdot\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}, \tag{10}$$

Δ^e is the area of the finite element, y_m is the center mass coordinate of the triangle.

We transform the second term from equation (6):

$$\delta\Pi_{b2}=\int_{A_b}\lambda_b\left(\varepsilon_x+\varepsilon_y\right)[1\quad y]dA\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}, \tag{11}$$

for a single finite element:

$$\delta\Pi^e_{b2}=\lambda_b\{b\}^T\left\{U^e_{pl}\right\}[1\quad y_m]\cdot\Delta^e\cdot\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}+$$

$$+\lambda_b\{c\}^T\left\{U^e_{pl}\right\}[1\quad y_m]\cdot\Delta^e\cdot\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}= \tag{12}$$

$$=\lambda_b\{g\}^T\left\{U^e_{pl}\right\}[1\quad y_m]\cdot\Delta^e\cdot\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}.$$

Here $\left\{U^e_{pl}\right\}$ is the nodal displacements vector of the finite element in the section plane, $\left\{U^e_{pl}\right\}=\left[u_i\quad v_i\quad u_j\quad v_j\quad u_k\quad v_k\right]^T$, where i, j, k are node's numbers, $\{b\}^T=\frac{1}{2\Delta^e}\left[b_i\quad 0\quad b_j\quad 0\quad b_k\quad 0\right]$, $\{c\}^T=\frac{1}{2\Delta^e}\left[0\quad c_i\quad 0\quad c_j\quad 0\quad c_k\right]$, $\{g\} = \{b\} + \{c\}$ the vector's $\{b\}$ and $\{c\}$ elements values b and c depend on the coordinates of the element nodes and are calculated similarly to the case of solving the plane problem of the theory of elasticity using the FEM [5].

Transforming expression (12) and transposing with multiple use of the property for the product of two matrices $\left([A][B]\right)^T=[B]^T[A]^T$, we get:

$$\delta\Pi^e_{b2}=\delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T\lambda_b\begin{Bmatrix}1\\ y_m\end{Bmatrix}\{g\}^T\cdot\Delta^e\cdot\left\{U^e_{pl}\right\}. \tag{13}$$

Denote as:

$\left[D_b\right]=\begin{bmatrix}\lambda_b+2\mu_b & \lambda_b & 0\\ \lambda_b & \lambda_b+2\mu_b & 0\\ 0 & 0 & \mu_b\end{bmatrix}$ is stress-strain elastic matrix, $\left\{\varepsilon_{pl}\right\}=\left[\varepsilon_x\quad\varepsilon_y\quad\gamma_{xy}\right]^T$ is elastic strain vector in the section plane. Then:

$$\delta\Pi_{b3}=\int_{A_b}\left\{\varepsilon_{pl}\right\}^T\left[D_b\right]\cdot\delta\left\{\varepsilon_{pl}\right\}dA \tag{14}$$

or after discretization:

$$\delta\Pi^e_{b3}=\left(\left[B_{pl}\right]\left\{U^e_{pl}\right\}\right)^T\left[D_b\right]\cdot\left[B_{pl}\right]\delta\left\{U^e_{pl}\right\}\cdot\Delta^e=$$

$$=\left(\left(\left[B_{pl}\right]\left\{U^e_{pl}\right\}\right)^T\left[D_b\right]\cdot\left[B_{pl}\right]\delta\left\{U^e_{pl}\right\}\right)^T\cdot\Delta^e= \tag{15}$$

$$=\left(\left[D_b\right]\left[B_{pl}\right]\delta\left\{U^e_{pl}\right\}\right)^T\left[B_{pl}\right]\left\{U^e_{pl}\right\}\cdot\Delta^e=$$

$$= \delta\left\{U_{pl}^{e}\right\}^{T}\left[B_{pl}\right]^{T}\left[D_{b}\right]^{T}\left[B_{pl}\right]\left\{U_{pl}^{e}\right\}\cdot\Delta^{e} =$$
$$= \delta\left\{U_{pl}^{e}\right\}^{T}\left[B_{pl}\right]^{T}\left[D_{b}\right]\left[B_{pl}\right]\cdot\Delta^{e}\cdot\left\{U_{pl}^{e}\right\},$$

$[D_b]^T = [D_b]$ due to the symmetry of the stress-strain elastic matrix, $[D_{pl}]$ is strain-displacement matrix:

$$\left[B_{pl}\right] = \frac{1}{2\Delta^{e}}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix}.$$

$$\delta\Pi_{b4} = \int_{A_b}\lambda_b\begin{Bmatrix}\varepsilon_0 + y\chi_0 \\ \varepsilon_0 + y\chi_0 \\ 0\end{Bmatrix}^{T}\cdot\delta\left\{\varepsilon_{pl}\right\}dA =$$
$$= \int_{A_b}\begin{Bmatrix}\varepsilon_0 \\ \chi_0\end{Bmatrix}^{T}\begin{bmatrix}1 & y \\ 1 & y \\ 0 & 0\end{bmatrix}^{T}\lambda_b\delta\left\{\varepsilon_{pl}\right\}dA. \quad (16)$$

Discretizing equation (16), we obtain:

$$\delta\Pi_{b4} = \begin{Bmatrix}\varepsilon_0 \\ \chi_0\end{Bmatrix}^{T}\begin{bmatrix}1 & y_m \\ 1 & y_m \\ 0 & 0\end{bmatrix}^{T}\lambda_b\left[B_{pl}\right]\cdot\Delta^{e}\cdot\delta\left\{U_{pl}^{e}\right\} =$$
$$= \delta\left\{U_{pl}^{e}\right\}^{T}\left[B_{pl}\right]^{T}\begin{bmatrix}1 & y_m \\ 1 & y_m \\ 0 & 0\end{bmatrix}\lambda_b\cdot\Delta^{e}\cdot\begin{Bmatrix}\varepsilon_0 \\ \chi_0\end{Bmatrix}. \quad (17)$$

Thus:

$$\delta\Pi_{b}^{e} = \delta\Pi_{b1}^{e} + \delta\Pi_{b2}^{e} + \delta\Pi_{b3}^{e} + \delta\Pi_{b4}^{e}, \quad (18)$$

the terms $\delta\Pi_{bi}^{e}$ are determined for each plane finite element using expressions (10), (13), (15), (17). The concrete core total potential energy variation is calculated by summing over the elements:

$$\delta\Pi_{b} = \sum_{e}\delta\Pi_{b}^{e}. \quad (19)$$

Let $[K]$ be the CFST column stiffness matrix of dimension $(2n + 2) \times (2n + 2)$, n is the number of nodes in the section, in each of which there are 2 degrees of freedom, 2 is the number of additional degrees of freedom (ε_0 and χ_0), that determine the column behavior along z axis direction. Then:

$$\left[K\right] = \left[K_b\right] + \left[K_s\right] = \sum\left[K_b^{e}\right] + \sum\left[K_s^{e}\right], \quad (20)$$

$[K_b]$, $[K_S]$ are matrices that determine the contribution of concrete and steel to the total stiffness matrix. Due to the fact that equality (1) must be valid for arbitrary values of variations, for a triangular concrete finite element we get:

$$\left[K_b^{e}\right] = \begin{bmatrix}\left[K_{b11}^{e}\right] & \left[K_{b12}^{e}\right] \\ \left[K_{b21}^{e}\right] & \left[K_{b22}^{e}\right]\end{bmatrix}, \quad (21)$$

$\left[K_b^{e}\right]$ is the 8 × 8 finite element stiffness matrix. A vector that determines the displacements of the *FE* with nodes i, j, k:

$$\left\{U^{e}\right\} = \left[\varepsilon_0 \quad \chi_0 \quad u_i \quad v_i \quad u_j \quad v_j \quad u_k \quad v_k\right]^{T},$$

the values ε_0 and χ_0 are common to all elements. The submatrices included in the composition $\left[K_b^{e}\right]$ in accordance with (10), (13), (15), (17) will be equal:

$$\left[K_{b11}^{e}\right] = \left(\lambda_b + 2\mu_b\right)\begin{bmatrix}1 & y_m \\ y_m & y_m^2\end{bmatrix}\cdot\Delta^{e},$$

$$\left[K_{b12}^{e}\right] = \lambda_b\begin{Bmatrix}1 \\ y_m\end{Bmatrix}\left\{g\right\}^{T}\cdot\Delta^{e},$$

$$\left[K_{b21}^{e}\right] = \left[B_{pl}\right]^{T}\begin{bmatrix}1 & y_m \\ 1 & y_m \\ 0 & 0\end{bmatrix}\lambda_b\cdot\Delta^{e} = \left[K_{b12}^{e}\right]^{T},$$

$$\left[K_{b22}^{e}\right] = \left[B_{pl}\right]^{T}\left[D_b\right]\left[B_{pl}\right]\Delta^{e}.$$

Total nodal force vector:

$$\left\{P\right\} = \left[-F \quad -M \quad 0 \quad 0 \quad \cdots \quad 0\right]^{T},$$

with dimensions $(2n + 2) \times 1$.

We model the steel tube with 1D bar finite elements with two translational degrees of freedom in the section plane at each of the nodes (Fig. 2).

The tube has a smooth shape. It is loaded with pressure from the side of the concrete core, continuously and smoothly changing along the circumference of the inner face, therefore, outside the fixed points there is a momentless biaxial stress state of the shell, the rotational degrees of freedom in the nodes of the steel bar FE are therefore not taken into account.

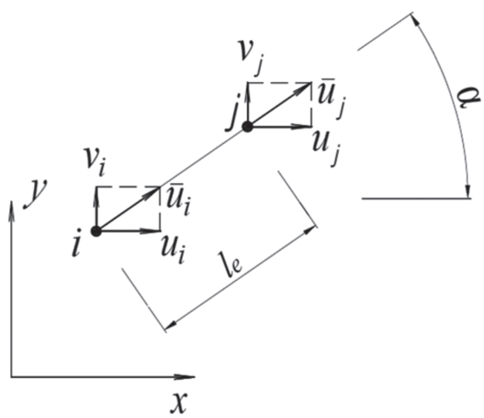

Fig. 3. 1D bar finite element

Strain potential energy of steel tube variation:

$$\delta\Pi_S = \int_{A_s} \left(\sigma_z \delta\varepsilon_z + \sigma_\theta \delta\varepsilon_\theta\right) dA, \quad (22)$$

A_s is area of the steel part of cross-section. Radial stress in the tube $\sigma_r \approx 0$ due to the small steel thickness $t_s \ll D_b$.

Using the generalized Hooke's law and taking into account the equality of radial stresses to zero, we obtain:

$$\sigma_z = E_s\varepsilon_z + \nu_s\left(E_s\varepsilon_\theta + \nu_s\sigma_z\right) \Rightarrow \sigma_z = = \frac{E_s}{1-\nu_s^2}\varepsilon_z + \frac{\nu_s E_s}{1-\nu_s^2}\varepsilon_\theta. \quad (23)$$

We denote $\tilde{E}_s = \frac{E_s}{1-\nu_s^2}$, then the expressions for the relationship between normal stresses and strains will take the form:

$$\sigma_z = \tilde{E}_s\varepsilon_z + \nu_s\tilde{E}_s\varepsilon_z. \quad (24)$$

the equation for the circumferential stress is obtained similarly by the permutation of the indices:

$$\sigma_\theta = \tilde{E}_s\varepsilon_0 + \nu_s\tilde{E}_s\varepsilon_z. \quad (25)$$

Substituting (24), (25) in (22):

$$\delta\Pi_s = \int_{A_s} \tilde{E}_s\varepsilon_z\delta\varepsilon_z dA + \int_{A_s} \nu_s\tilde{E}_s\varepsilon_\theta\delta\varepsilon_z dA + \int_{A_s} \tilde{E}_s\varepsilon_\theta\delta\varepsilon_\theta dA + + \int_{A_s} \nu_s\tilde{E}_s\varepsilon_z\delta\varepsilon_\theta dA = \delta\Pi_{s1} + \delta\Pi_{s2} + \delta\Pi_{s3} + \delta\Pi_{s4}. \quad (26)$$

We transform the term $\delta\Pi_{s1}$ similarly to the term $\delta\Pi_{b1}$:

$$\delta\Pi_{s1} = \delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T \int_{A_s} \tilde{E}_s \begin{bmatrix}1 & y\\ y & y^2\end{bmatrix} dA \begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}, \quad (27)$$

For a single bar finite element with length l_e:

$$\delta\Pi_{s1}^e = \delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T \tilde{E}_s \begin{bmatrix}1 & y_m\\ y_m & y_m^2\end{bmatrix} l_e \cdot t_s \begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}, \quad (28)$$

$$\delta\Pi_{s2} = \delta\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}^T \int_{A_s} \begin{Bmatrix}1\\ y\end{Bmatrix} \nu_s\tilde{E}_s\varepsilon_\theta dA. \quad (29)$$

Let $\left\{\bar{U}_{tr}^e\right\} = \begin{Bmatrix}\bar{u}_i\\ \bar{u}_j\end{Bmatrix}$ be the nodal displacements of the bar finite element in the section plane in the local coordinate system, $\left\{U_{tr}^e\right\} = \begin{bmatrix}u_i & \nu_i & u_j & \nu_j\end{bmatrix}^T$ are in the global coordinate system, then

$$\left\{\bar{U}_{tr}^e\right\} = [L]\left\{U_{tr}^e\right\}, \quad (30)$$

where $[L]$ is directional cosine matrix, $[L] = \begin{bmatrix}r & s & 0 & 0\\ 0 & 0 & r & s\end{bmatrix}$, $r = \cos\alpha$, $s = \sin\alpha$. Then:

$$\varepsilon_\theta^e = \begin{bmatrix}-\frac{1}{l^e} & -\frac{1}{l^e}\end{bmatrix}\left\{\bar{U}_{tr}^e\right\} = [B_{tr}]\left\{\bar{U}_{tr}^e\right\} = [B_{tr}][L]\left\{U_{tr}^e\right\}. \quad (31)$$

Equation (29) for a single finite element takes the form:

$$\delta\Pi_{s2}^e = \delta\begin{Bmatrix}\varepsilon_0\\ \chi\end{Bmatrix}^T \nu_s\tilde{E}_s \begin{Bmatrix}1\\ y_m\end{Bmatrix} [B_{tr}][L]\left\{U_{tr}^e\right\}. \quad (32)$$

$$\delta\Pi_{s3}^e = \tilde{E}_s\left([B_{tr}][L]\left\{U_{tr}^e\right\}\right)\left([B_{tr}][L]\delta\left\{U_{tr}^e\right\}\right) l_e t_s. \quad (33)$$

Transposing (33), we get:

$$\delta\Pi_{s3}^e = \delta\left\{U_{tr}^e\right\}^T [L]^T [B_{tr}]^T [B_{tr}][L] \cdot l_e t_s \left\{U_{tr}^e\right\}. \quad (34)$$

$$\delta\Pi_{s4}^e = \delta\varepsilon_\theta^e \cdot \tilde{E}_s\nu_s \begin{bmatrix}1 & y_m\end{bmatrix}\begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix} l_e t_s = = \delta\left\{U_{tr}^e\right\}^T \tilde{E}_s\nu_s [L]^T [B_{tr}]^T \begin{bmatrix}1 & y_m\end{bmatrix} l_e t_s \begin{Bmatrix}\varepsilon_0\\ \chi_0\end{Bmatrix}. \quad (35)$$

The total strain potential energy variation of the steel tube calculated by summing over the elements:

$$\delta\Pi_s = \sum_e \delta\Pi_s^e = \sum_e \left(\delta\Pi_{s1}^e + \delta\Pi_{s2}^e + \delta\Pi_{s3}^e + \delta\Pi_{s4}^e\right). \quad (36)$$

Basing on (28), (32), (34), (35) and (36) for bar finite elements we obtain:

$$\left[K_s^e\right] = \begin{bmatrix} \left[K_{s11}^e\right] & \left[K_{s12}^e\right] \\ \left[K_{s21}^e\right] & \left[K_{s22}^e\right] \end{bmatrix}, \quad (37)$$

$\left[K_s^e\right]$ is FE stiffness matrix of dimension 6 × 6. Vector defining the FE displacements with nodes i, j: $\left\{U^e\right\} = \left[\varepsilon_0 \quad \chi_0 \quad u_i \quad \nu_i \quad u_j \quad \nu_j\right]^T$,

$$\left[K_{s11}^e\right] = \tilde{E}_s \begin{bmatrix} 1 & y_m \\ y_m & y_m^2 \end{bmatrix} l^e t_s,$$

$$\left[K_{s12}^e\right] = \tilde{E}_s \nu_s \begin{bmatrix} 1 \\ y_m \end{bmatrix} \left[B_{tr}\right]\left[L\right] l^e t_s,$$

$$\left[K_{s21}^e\right] = \tilde{E}_s \nu_s \left[L\right]^T \left[B_{tr}\right]^T \left[1 \quad y_m\right] l^e t_s = \left[K_{s12}^e\right]^T,$$

$$\left[K_{s22}^e\right] = \left[L\right]^T \left[B_{tr}\right]^T \left[B_{tr}\right]\left[L\right] \cdot l^e t_s.$$

Using equation (20), we find the stiffness matrix [K]. The problem is reduced to solving a system of linear equations:

$$\left[K\right]\left\{U\right\} = \left\{P\right\}, \quad (38)$$

where $\left\{U\right\} = \left[\varepsilon_0 \quad \chi_0 \quad u_1 \quad \nu_1 \quad u_2 \quad \cdots \quad u_n \quad \nu_n\right]^T$ is nodal displacements vector, n is the number of nodes in the section.

Method of calculation

System (38) will be composed and solved using the MATLAB software package. An example of meshing using the «initmesh()» function is shown in Fig. 4. The concrete core diameter is 0.2 m here, node numbers are shown in black.

The stiffness matrix and the load vector composing is performed using the formulas specified in section 2 of the article.

The stresses are calculated after determining the nodal displacements according to the formulas, obtained from the generalized Hooke's law for concrete and steel elements:

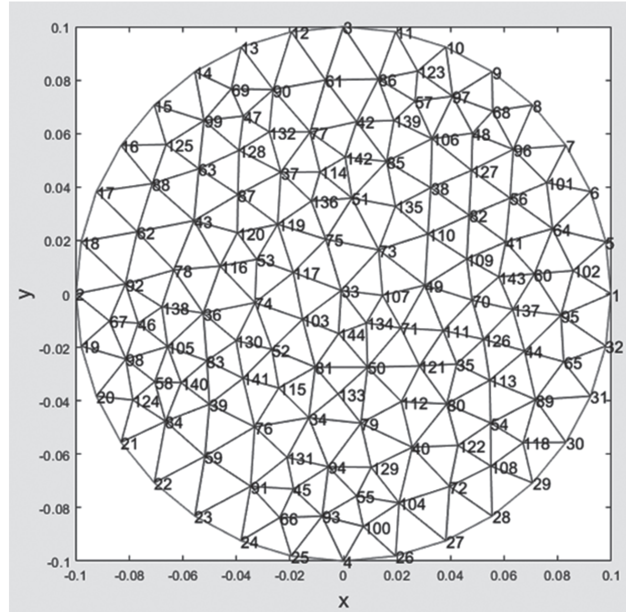

Fig. 4. Finite element mesh made with «initmesh()» function

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^e = \left[D_b\right]\left[B_{pl}\right]\left\{U_{pl}^e\right\} + \lambda_b \begin{Bmatrix} \varepsilon_0 + y_m\chi_0 \\ \varepsilon_0 + y_m\chi_0 \\ 0 \end{Bmatrix},$$

$$\sigma_{zb}^e = E_b\left(\varepsilon_0 + y_m\chi_0\right) + \nu_b\left(\sigma_{xb}^e + \sigma_{yb}^e\right),$$

$$\sigma_{\theta s}^e = \tilde{E}_s\left[B_{tr}\right]\left[L\right]\left\{U_{tr}^e\right\} + \nu_s \tilde{E}_s\left(\varepsilon_0 + y_m\chi_0\right).$$

The obtained results will be compared with the results of calculations in the ANSYS software using volumetric finite elements for concrete and shell ones for a steel tube.

Results and discussion

When solving the test problem, we will use the following initial data: d_b = 0,2 m, E_b = 3 · 10[4] MPa, E_s = 2 · 10[5] MPa, ν_b = 0,2, ν_s = 0,3, t_s = 0,008 m, F = 1 MN, M = 0,1 $MN \cdot m$, the column length is taken equal 1 m. The loaded model of the problem when solving in the ANSYS is shown in Fig. 5. The obtained results are shown in Fig. 6–7.

The results of calculations in the MATLAB software package according to the technique presented in the article by reducing the problem to two-dimensional are shown in Fig. 8–9.

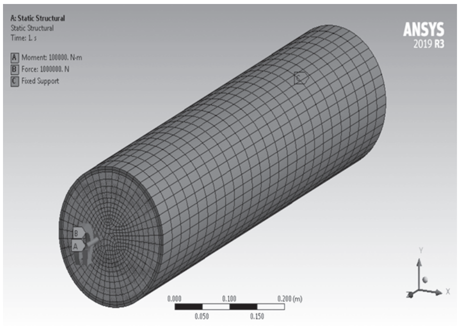

Fig. 5. The finite element mesh used in the simulation in ANSYS

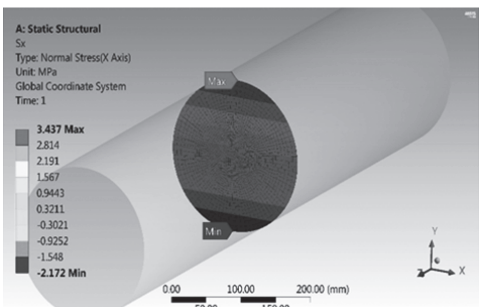

Fig. 6. Concrete stress σ_x distribution in the middle section of the CFST column (ANSYS)

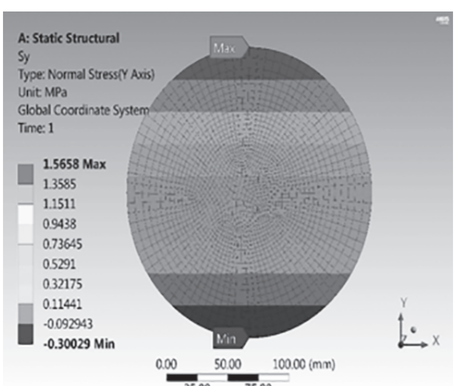

Fig. 7. Concrete stress σ_y distribution in the middle section of the CFST column (ANSYS)

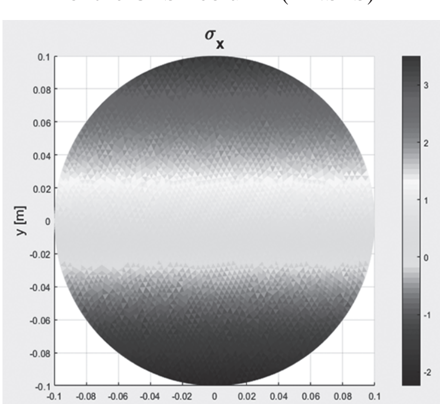

Fig. 8. Concrete stress σ_x distribution in the section of the CFST column (MATLAB)

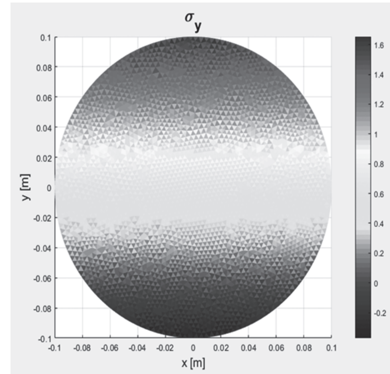

Fig. 9. Concrete stress σ_y distribution in the section of the CFST column (MATLAB)

The maximum values of σ_x in concrete as calculated in ANSYS and MATLAB were 3,44 *MPa* and 3,50 *MPa*, respectively. The error is about 1,7%. The maximum normal stresses in concrete were 1,57 *MPa* and 1,65 *MPa*, respectively.

It is important to note that a simplified elastic model of concrete was used in the calculations, the material physical nonlinearity, as well as the dilatation effect, are not taken into account. As a result, the negative stresses of compression from the side of the tube are insignificant and are located in a smaller part of the concrete section.

Conclusions

In the presented article, a method has been developed for reducing the problem of determining the stress-strain state of short CFST columns to two-dimensional. Expressions for the stiffness matrices elements of the column are obtained. To implement the algorithm described in the work, a program was written in MATLAB. The results obtained are compared with the results of calculations in the modern finite element software ANSYS by modeling the deformation of concrete with volumetric finite elements, and the steel tube with shell ones. The resulting error is within acceptable limits, while the use of plane finite elements together with bar ones according to the presented method significantly reduces the order of the FEM system of equations and requires less resources.

In the calculations, the concrete was modeled as linearly elastic. Using the proposed approach, it is possible to solve the problem in a physically nonlinear formulation, including taking into account creep. When solving creep problems, the approaches presented in [6–10] can be used.

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